

ABSTRACT

The application of multiobjective optimization is currently receiving growing interest from researchers with various backgrounds. Most research in this area has understandably concentrated on the selection stage, due to the need to integrate vectorial performance measures with the inherently scalar way in which multi objective reward individual performance. In this review, current multiobjective approaches are discussed, ranging from the conventional analytical aggregation of the different objectives into a single function to a number of population-based approaches and the more recent ranking schemes based on the definition of optimality. The sensitivity of different methods to objective scaling and/or possible concavities are considered. From the discussion, directions for future research in multiobjective fitness assignment and search strategies are identified, including the incorporation of decision making in the selection procedure, fitness sharing, and adaptive representations

KEYWORDS: Multi-Objective Optimization; Intelligent Optimization Algorithms; Trade off methods

I. INTRODUCTION

1. Multi-objective optimization algorithms

Optimization problems are mainly solved by two methods [1]: analytical method and numerical method. The analytical method involves strict mathematical proofs and derivation, and it can reach exact solution. However, the method is also strict with the problem characteristics, which many realistic problems could not match. The numerical method is designed with appropriate iteration formulas and applied with a series of iterations to get the approximate solution. It needs only defined decision variables and objective variables feedback from optimization problems. The optimization problem can be a black-box problem without obvious expressions and it is more suitable for realistic problems. Among the numerical methods [2], classical methods like Newton iteration method, simplex method, conjugate direction method, etc. are oriented at single objective optimization problems.

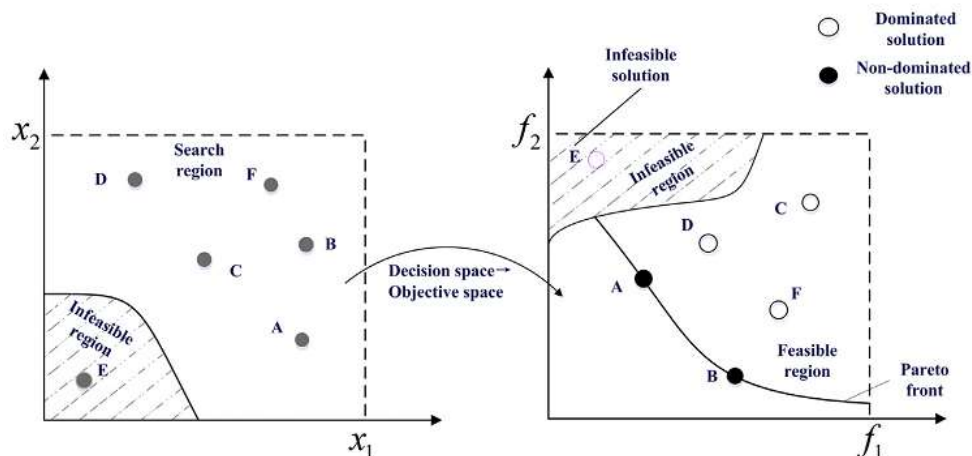


Fig. 1 .Relationship between the design space and the objective space and solution definition of a two-objective problem

These methods have high searching efficiency and fast convergence speed, because they usually start with a given initial point and calculate the next iteration point according to the descending information such as gradient. However, the methods have difficulties in solving problems whose gradient information cannot or need too much to be calculated. When applied for solving multimodal objective functions, classical numerical methods are easy to fall into and hard to escape from local optimum solutions. Moreover, the methods find only one single solution in each iteration step, and the accuracy of the solution depends mostly on the setting of initial values. Intelligent numerical methods, as one sort of heuristic search algorithm, are inspired by behaviors, reactions and communication mechanisms in nature. Thus developed optimization algorithms are broadly divided into four categories [3] as shown in Fig.2: Biology inspired algorithms, Physics inspired algorithms, Geography inspired algorithms, and Social culture inspired algorithms.

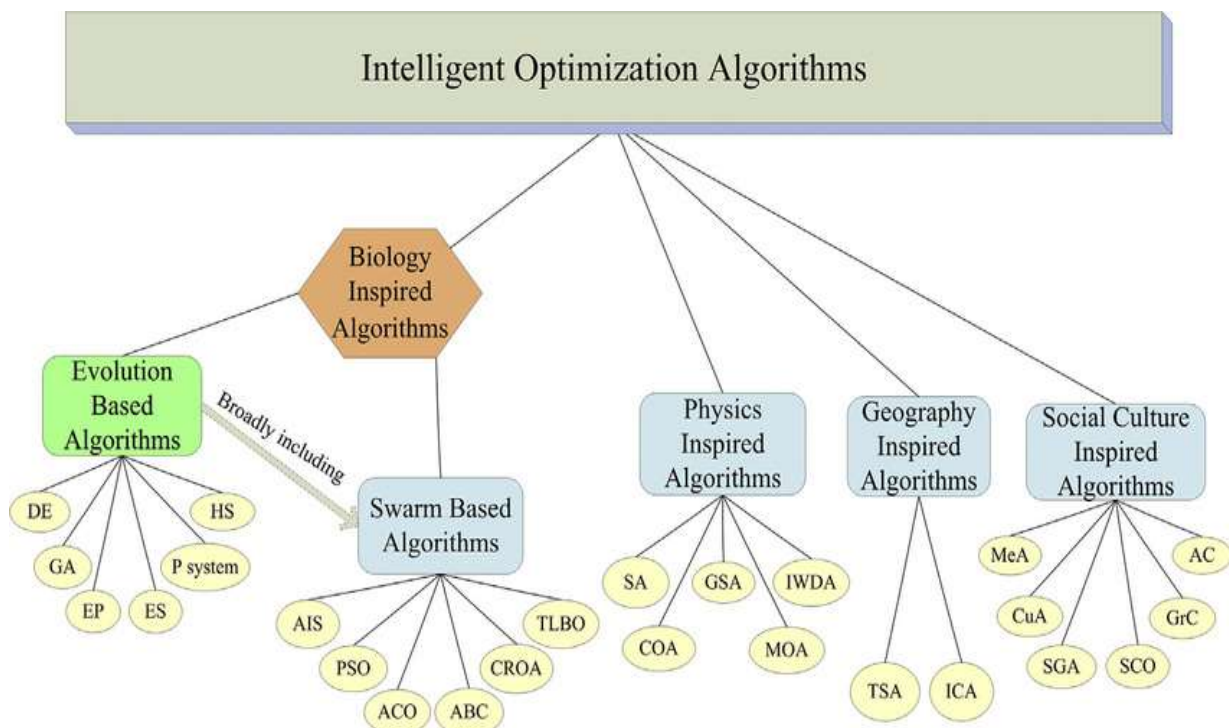


Fig.2. Classification of Intelligent Optimization Algorithms

Biology inspired algorithms

Biology inspired algorithms are inspired from biological activities in both micro and macro world (such as evolution behaviors), or from substantial development and structural features [4]. They are generally divided into two types: evolution based algorithms and swarm based algorithms [5]. (i) Evolution based algorithms Evolution based algorithms, also known as Evolutionary Algorithms (EA) are stochastic search methods that mimic the survival of the fittest process of natural ecosystems. The algorithms have strong adaptability and self-organization, including Evolutionary Programming (EP) [6], Evolutionary Strategy (ES) [7], Genetic Algorithm (GA), Differential Evolution Algorithm (DE), Harmony Search Algorithm (HS), Membrane Computing (P system), etc. [8]. The development process of classical genetic algorithms and differential evolution algorithms are respectively demonstrated in Table 1 and Table 2.

Table 1
 Important developments of genetic algorithms.

Genetic algorithms developments	Initially proposed time	Proposers	Run-time complexity
GA	1975	Holland	
VEGA (vector-evaluated genetic algorithm)	1985	Schaffer	
MOGA (Multiobjective Genetic Algorithm)	1993	Fonseca and Fleming	$O(GmN^2)$
NSGA (Non-dominated Sorting Genetic Algorithm)	1993	Srinivas and Deb	$O(GmN^2)$
NPGA (Niche Pareto Genetic Algorithm)	1994	Horn and Nafpliotis	$O(GmN^2)$
SPEA (Strength Pareto Evolutionary Algorithm)	1999	Zitzler and Thiele	$O(Gm(N + A)^2)$
PAES (Pareto Archived Evolution Strategy)	2000	Knowles and Corne	$O(GN \log^{M-1} A \log \log A)$
PESA (Pareto Envelope-Based Selection Algorithm)	2000	Corne, Knowles and Oates	$O(GN \log^{M-1} A \log \log A)$
PESA-II	2001	Knowles, Jerram, Corne, et al.	$O(GmNA)$
NPGA2	2001	Erichson et al.	
Micro-GA	2001	Coello Coello et al.	
SPEA2	2002	Zitzler, Laumanns and Thiele	$O(Gm(N + A)^2)$
NSGA-II	2002	Deb et al.	$O(GN \log^{M-1} N)$

Table 2
 Variants of differential evolution algorithms.

Differential evolution algorithms	Initially proposed time	Proposer
DE	1995	Storn and Price
FADE (adaptive differential evolution algorithm based on fuzzy theory)	2005	Liu and Lampinen
SaDE (scaling factor adaptive adjustment differential evolution)	2009	Qin et al.
MOSaDE (multi-objective SaDE)	2009	Huang

Moreover, Jia et al. [9] added chaos neighborhood searching mechanism to DE to improve its search ability in the early search stage and exploration ability in the later search stage. Liu et al. [10] combined DE with PSO to form a hybrid algorithm, which improved the performance and accelerated the searching efficiency. The hybrid algorithm worked especially well for solving constrained optimization problems. Membrane computing [11], also known as a P system, is non-deterministic and distributed parallel computing device, which is abstracted from the structure and functioning of living cells, as well as from the interactions of living cells in tissues or neuros. It was initiated by Păun in 1998, with the first paper published in 2000 [12]. The structure is consisted of several cell-like membranes, placed inside a solo skin membrane. Multisets of objects are placed in the regions delimited by hierarchical or more general arrangements of membranes, as shown in Fig.3. The evolution processes of each object are done in a parallel manner. At last, the evolved result is output from the skin membrane to the environment. There are mainly three types of P systems: cell-like P systems; tissue-like P systems and neural-like P systems [13].

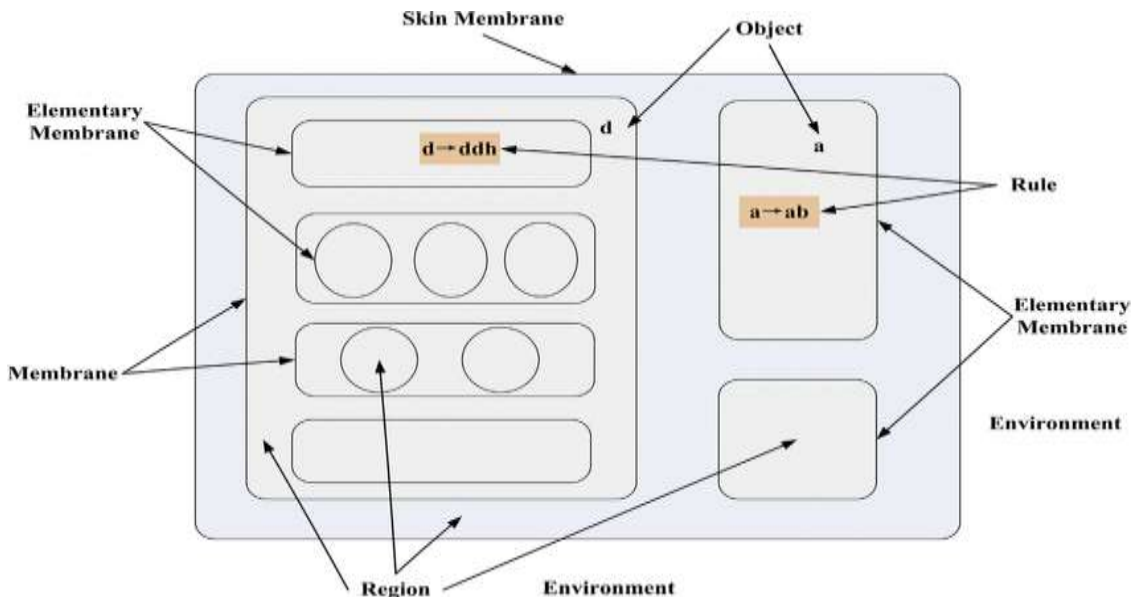


Fig.3. A Membrane Structure

It has been proved that any Turing computable problems can be solved by P systems. The P system has already applications in computer graphics, computer science, cryptography, mathematics, abstract chemistry, biology, ecology, artificial intelligence, approximate optimization, and even linguistics, etc. HS was originally developed by Geem [14] for discrete-variable problems and then expanded to continuous-variable problems. It mimics musician's behaviors such as random play, memory-based play and pitch adjusted play to get a perfect state of harmony. Wang et al. [15] proposed a differential harmony search (DHS) algorithm, combining the mechanisms of differential evolution with harmony search. The DHS enhances the exploration ability of the algorithm. (ii) Swarm based algorithms In a broadly defined way, swarm based algorithms are included in evolution based algorithms. Swarm based algorithms are inspired from social nature and model the collective behavior of populations, such as honey bees, ant colonies, and bird flocks, etc. Among these agents (swarm individuals), they cooperate with each other to search for food, necessary for their survival, and also keep safe from other agents. Swarm based algorithms are consisted of Particle Swarm Optimization Algorithm (PSO), Artificial Bee Colony Algorithm (ABC) [16], Artificial Immune System (AIS), Teaching-Learning Based Optimization algorithm (TLBO), Ant Colony Optimization Algorithm (ACO) [17], Cuckoo Search algorithm (CS), Firefly Algorithm (FA) [18], Bacteria Foraging Optimization algorithm (BFO) [19], Coral Reef Optimization algorithm (CROA) [20], Shuffled Frog Leaping Algorithm (SFLA)[21], Pigeon Inspired Optimization (PIO) [22], etc. In 1987, Christopher initially proposed the concept of Artificial Life, which meant the system that mimic the behavior characteristics in the nature life system, by the way of computers or other non-biological media [23, 24]. The above mentioned evolution-based algorithms are inspired by Darwinian evolution whereas the swarm intelligence is generated by imitating the behaviors of social swarms [25]. Swarm intelligence [26] meant the intelligent behaviors presented by simple behaviors of individuals in the population without central control. These individuals behaved to solve the foraging, searching and visiting, transportation and transmission problems. Based on the swarm intelligence, swarm-based algorithms were then produced.

(a) Particle Swarm Optimization (PSO)

In 1995, Kennedy and Eberhart [27] proposed the PSO algorithm, whose central idea was information sharing mechanism. The PSO has been widely used in various fields to solve different kinds of optimization problems. Many variants of the PSO algorithm have been proposed to maintain or strengthen the diversity, so as to escape from local optima or premature convergence. Li et al. [28] combined PSO with NSGA-II and the experiment results showed that the combined method had better performance than NSGA-II. Coello Coello et al. [29] proposed the multi-objective PSO (MOPSO), which incorporated external population with adaptive grids.

(b) Teaching-learning based optimization (TLBO)

The TLBO algorithm was first proposed by Rao et al. [30]. There are two phases in the TLBO: teacher phase and learner phase. Learners learn from the teacher in the teacher phase and from each other in the learner phase. The teacher is considered as the best solution in the entire population obtained thus far. In order to get the global optimal solutions, a modified TLBO

algorithm was proposed by Hosseinpour et al. [31], which added a mutation process similar to that of DE. Niknam et al. [32] proposed a modified TLBO algorithm with two mutation operations and two crossover operations added, so as to enhance the local and global search abilities of the algorithm. TLBO also shows good performance in solving large scale optimization problems with little computational efforts [33].

(c) Artificial Immune System (AIS)

The AIS is inspired from immunology and acts as an adaptive system that mimics the function, principles and model of immunology to solve complicated problems [34]. It has been successfully applied in fields of anomaly detection, computer security, data mining, optimization, etc. In Table 3 illustrated are developments of AIS. Wherein, the NNIA is especially advantageous in solving many-objective optimization problems with more than three optimization objectives. Compared to classical numerical methods, broadly defined evolution based algorithms is one sort of probability search algorithm based on population. These algorithms need neither extra initial points nor gradient information of objective functions. Therefore, they are suitable for optimization problems that cannot be solved by classical numerical methods. Moreover, evolution based algorithms have characteristics of parallelism and distribution, appropriate for solving large-scale/ high-dimensional optimization problems.

Table 3

Developments of the artificial immune system.

Developments of the artificial immune system	Initially proposed time	Proposer
AIS	1998	Dasgupta
MISA (multi-objective immune system algorithm)	2005	Coello Coello et al.
NNIA (non-dominated neighborhood immune algorithm)	2008	Gong et al.

Physics inspired algorithms

Physics inspired algorithms for optimization problems are also heuristic algorithms. They imitate the physical behaviors and properties of the matters or follow the same philosophy as the laws of physics. The common physics inspired algorithms include Chaotic Optimization Algorithm (COA), Intelligent Water Drops Algorithm (IWD) [35], Magnetic Optimization Algorithm (MOA) [36], Gravitational Search Algorithm (GSA) [37], Simulated Annealing (SA) [38], etc.

(a) Chaotic Optimization Algorithm (COA)

The random search technique was introduced by Hamzacebi and Kutay [39], which was adaptable to different optimization problems and the simplest heuristic algorithms. As introduced by Vela'squez Henao [40], the use of chaotic sequences instead of quasi-random numbers seemed to be a more powerful strategy for improving many traditional heuristic algorithms, because the chaotic sequences have characteristics of ergodicity, randomness, and regularity. An essential feature of chaotic systems is that small changes in the parameters or the initial values lead to vastly different future behaviors [41]. The chaos optimization algorithm (COA) was first proposed in 1997 by Li et al. [42], in which the Logistic map was introduced to produce chaos variables as optimization variables. Besides Logistic map, other mapping methods were also incorporated in the COA, such as Tent map [43]. The COA was also combined with other algorithms to form hybrid chaos optimization methods [44].

Geography inspired algorithms

Geography inspired algorithms are one sort of metaheuristic algorithm and generate random solutions in the geographical search space. These optimization algorithms are classified as Tabu Search Algorithm (TS), Imperialistic Competition Algorithm (ICA), etc.

(a) Imperialistic Competition Algorithm (ICA)

The ICA is inspired by imperialism, which is the policy of extending power and signifies the role of a government [45]. The number of colonies determines the power of an imperialist. Strengthening the authority of an imperialist makes other imperialists weaker. (b) Tabu Search Algorithm (TS) Tabu search algorithm was first suggested by Glover [46] in 1986, which was a meta-heuristic search method based on local search. It explores all feasible solutions in the search space by a sequence of moves. Especially, a set of moves are forbidden at each iteration step to escape from local minima [47].

Social culture inspired algorithms

Social culture inspired algorithms are inspired by the social, economic and cultural systems etc. that incorporate the cultural evolution theory into optimization algorithms. As shown in Table 4, there are some classical developments of social culture inspired algorithms. In 1989, Moscato firstly proposed the Memetic Algorithm, which used the local heuristic search to imitate the mutation process backed up by large amount of professional knowledge. The Granular Computing mimics the human thoughts from different levels of granules. It is based on the space partition of problem concepts, able to effectively analyze and deal with problems of fuzziness, non-accuracy, non-consistency and partial true values.

Table 4
 Developments of social culture inspired algorithms.

Social culture inspired algorithms	Source of inspiration	Initially proposed time	Proposer
MeA (Memetic Algorithm)	Social culture	1989	Moscato
CuA (Cultural Algorithm)	Social culture (signal, knowledge, etc.)	1994	Reynolds
SCO (Social Cognitive Theory)	Social cognitive process	2002	Xie
SGA (Selfish Gene Algorithm)	Selfish gene in human beings	1998	Corno et al.
GrC (Granular Computing)	Information cognition	1996	Lin
AC (Affective Computing)	Social and cultural affection, etc.	1997	Picard

In summary, Fig.4 demonstrates the overall development process of common intelligent optimization algorithms. At the same time, description of advantages and disadvantages of some classical intelligent optimization algorithms are listed in Table 5.

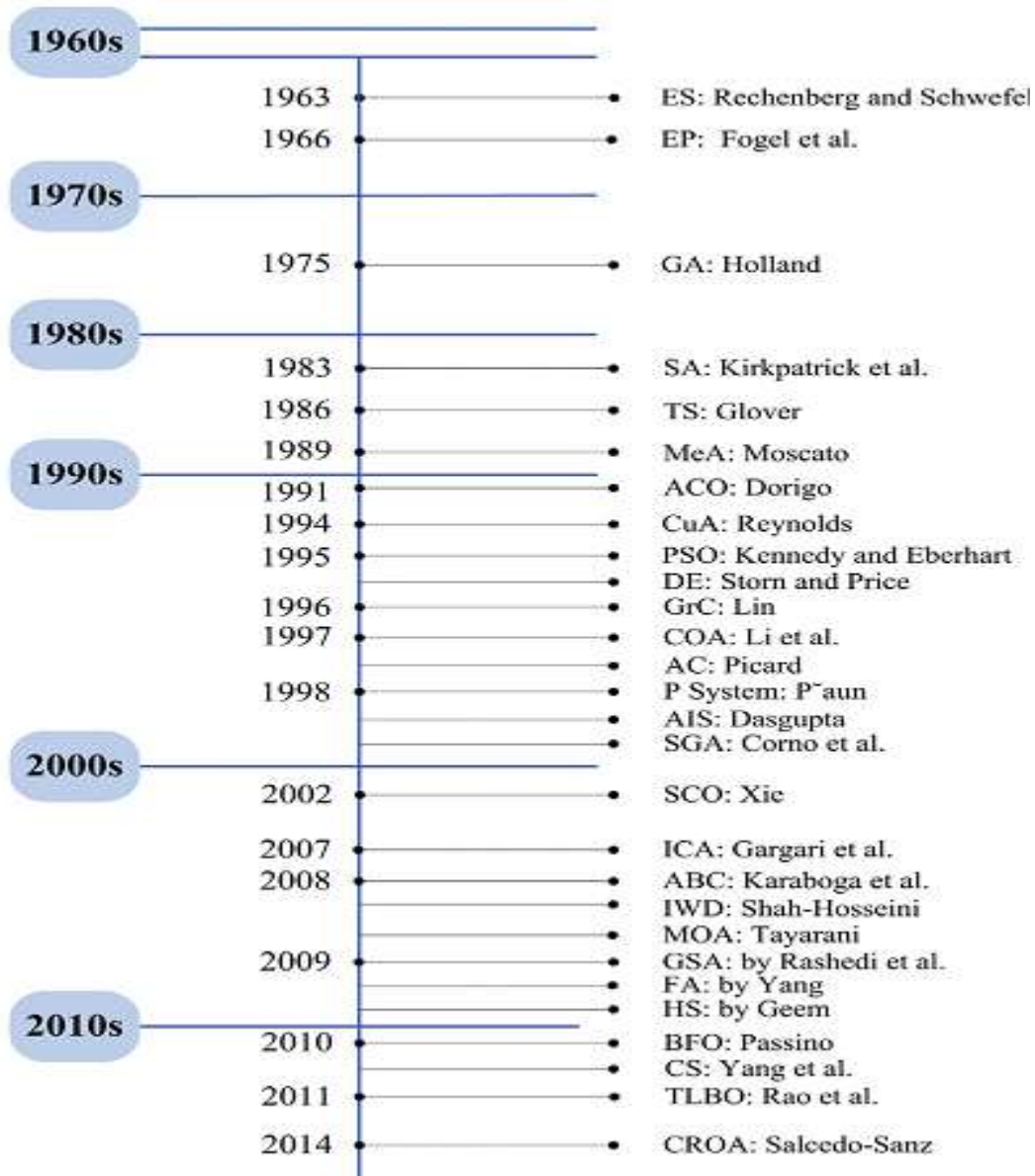


Fig.4. Development process of common intelligent optimization algorithms



(Dis)advantages of some classical intelligent optimization algorithms.

Intelligent optimization algorithms	Advantages	Disadvantages
GA(NSGA-II)	Good diversity of solutions (using the environment selection strategy based on neighborhood rules (in SPEA2), or the elitist keeping mechanism); Uniform distribution of Pareto solutions with good performance (based on the crowding distance method); Relatively low computation complexity (fast non-dominated solution ranking method).	Premature or long time convergence; Probable loss of best solutions; Not guaranteed to find a global optimum.
DE	Easy to understand and strong robustness; Few parameters need to be adjusted; Able to tackle non-differentiable, non-linear and multimodal functions promptly.	Its parameters need to be preset; Parameters have important influences on the algorithm performance.
P System	Understandable, scalable and programmable; Parallel execution manner; Adequate for handling discrete processes.	A relatively new and developing method, not mature in application.
HS	Only a few parameters and easy to be implemented.	Slow convergence speed and weak search guidance.
PSO	Simple procedures and not complex adjustment; Easy to be implemented with relatively fast speed; Able to escape from the local optima and find the global optimal solutions in most cases.	Not sure to be strictly convergent; Relatively weak local search ability; Probable to be immersed in local optima in multi-modal problems.
TLBO	Good learning abilities and parameter-free. Good performance in solving large scale optimization problems with little computational efforts.	Relatively low local search ability; Premature convergence due to lack of sufficient information sharing; Easy to be immersed in local optima when dealing with low dimension and complicated solution space.
AIS	Distributed, self-organizing and self-adaptable structure; Parallel and robust in execution; High search efficiency; Able to escape from premature convergence; Suitable for dealing with problems with constraints and multiple criteria.	Not profound or mature research and application, especially in the optimization field.
GSA	Global search, easy execution and implicit parallelism.	Need to be improved in search efficiency and solution precision.
SA	Simple structure, parallel computing, high efficiency, extensive applications and flexible to use; Can converge asymptotically to the global optimal solution; Especially suitable for solving large-scale combinatorial optimization problems.	Slow convergence; time-consuming in computation; Sensitive to parameters; May be constrained by initial conditions.
COA	Simple structure, easy implementation; Short execution time, high execution efficiency; Excellent stochastic searching capability for global optima and robust mechanisms of escaping from local optima.	Dependent on initial values; May not be able or need too much time to find or approximate the optimal solution under unsuitable initial values.
ICA	High convergence accuracy and fast convergence speed; Appropriate for optimization of nonlinear optimization problems with high dimensions.	Relatively low solution precision; Easy to be premature convergent and immersed in local optima.
TS	A global optimization algorithm; Able to escape from local optima; Effective in solving combinatorial optimization problems.	Dependent on initial solutions; Serial but not parallel computation process.
MeA	Combined priorities of global and local search; High search efficiency, fast convergence speed, parallel working manner; Relatively good diversity and fault tolerance ability.	The design of local search module has important influence on solutions.

Zhou et al. [48] surveyed the development of MOEAs (multi-objective EAs) in detail. In the paper covered included algorithmic frameworks and applications such as MOEAs with specific search methods, MOEAs for multimodal problems, constraint handling in or with MOEAs, computationally expensive multi-objective optimization problems (MOPs), dynamic MOPs, combinatorial and discrete MOPs, and etc. There was also a summary of the major applications of MOEAs in solving real-world problems. Rodrigues et al. [49], Cambero et al. [50], Chaouachi [51], H.A. et al. [52], and Fadaee et al. [53] reviewed the application of intelligent algorithms (especially EAs) in the multi-objective optimization of economic, energy, environment, or technical issues in the fields of wind farm, forest biomass supply chains, micro-grid, distribution generation systems, and hybrid renewable energy systems. They found out that intelligent algorithms were utilized effectively able to find global optima.

The PSO algorithm was one of the most popular intelligent algorithms that were applied for solving multi-objective optimization problems considering energy, economics and environment issues in processes that



produced or consumed energy [54]. As renewable energy resources are clean and environment-friendly, they are paid enormous attention in recent years. PSO [55] or its improved variants like MOPSO [56], AMOPSO [57], BB-MOPSO [58], and LAPSO [59] etc. were tested efficient in energy optimization applications. Meanwhile, GA, with most accepted and applied variants (including VEGA, MOGA, SPEA, SPEA2, NSGA, NSGA-II, PESA, PESA-II, NPGA, NPGA2, etc.) were also used or improved varying with concrete multi-objective optimization problems, like building energy optimization [60, 61], distribution transformer optimal design [62], complex industrial processes [63, 64], or processes integrated renewable resources [65].

II. MULTI-OBJECTIVE OPTIMIZATION TEST FUNCTIONS AND PERFORMANCE EVALUATION INDEXES

1. Multi-objective optimization test functions

As it is hard to evaluate the performance parameters of intelligent algorithms theoretically, researchers generally use test functions to verify the algorithm performances. Zitzler et al. [66] constructed a set of test problems called ZDT test function set, which was consisted of six problems ZDT1~ZDT6. These problems are two-objective optimization problems with different forms of expression and properties. Because their Pareto fronts are known, they are one of the most common used test problems. Therein, ZDT1 and ZDT4 are convex functions while ZDT2 and ZDT6 are concave functions, ZDT3 is a non-continuous function, ZDT4 is a multi-modal function, and ZDT5 is a function with deceptive property. Deb et al. [67] constructed a set called DTLZ testfunction set, which allowed the decision variables and objective functions to extend to any dimension. The DTLZ test function set includes seven unconstrained optimization problems DTLZ1~DTLZ7 and two constrained optimization problems DTLZ8~DTLZ9. They are also widely used for testing the performance of optimization algorithms. Deb et al. [68] also constructed a set of constrained multi-objective optimization problems (called DEB) with different Pareto optimal boundaries. Huband et al. [69] defined a set of WFG test problems, and constructed a scalable toolkit of test problems. Li and Zhang [70] proposed a set of continuous test problems whose variables were correlative and the Pareto front surface was with arbitrary complexity, which was able to reflect the complexity in the real-world multi-objective optimization problems. There are also some other common test problems like Schaffer's study (SCH) [71], Fonseca and Fleming's study (FON) [72], Kursawe's study (KUR) [73], etc.

2. Multi-objective optimization performance evaluation indexes

The solutions found by the multi-objective optimization algorithms are a set of approximate Pareto optimal solutions and we need to evaluate this set of approximate solutions. The evaluation indexes usually involve the following three indexes:

- (1) Convergence: the solutions that are most approximate to the Pareto optimal solutions are the best.
- (2) Uniformity: the good solutions should be distributed uniformly along the Pareto optimal frontier.
- (3) Distribution: the final solutions should cover the whole Pareto optimal frontier as much as possible.
- (4) **Multi-objective Optimization Trade-off Methods**: In order to get a trade-off solution for multiple, conflicting, and non-commensurate objectives, more and more researches have been done from classical optimization algorithms to intelligent optimization algorithms. In the past, the earliest and direct method for dealing with multi-objective problems was to transfer them into single objective problems, and then used classical optimization algorithms to solve the problems. In applying this method, we need decide the importance degree of each objective, which is previously determined using a priori method (like Weighted Sum Method) or determined during the search process using the interactive method (like Boundary Intersection Method). As shown in Fig.5, there is a summarized demonstration of trade-off methods for solving multi-objective optimization problems.

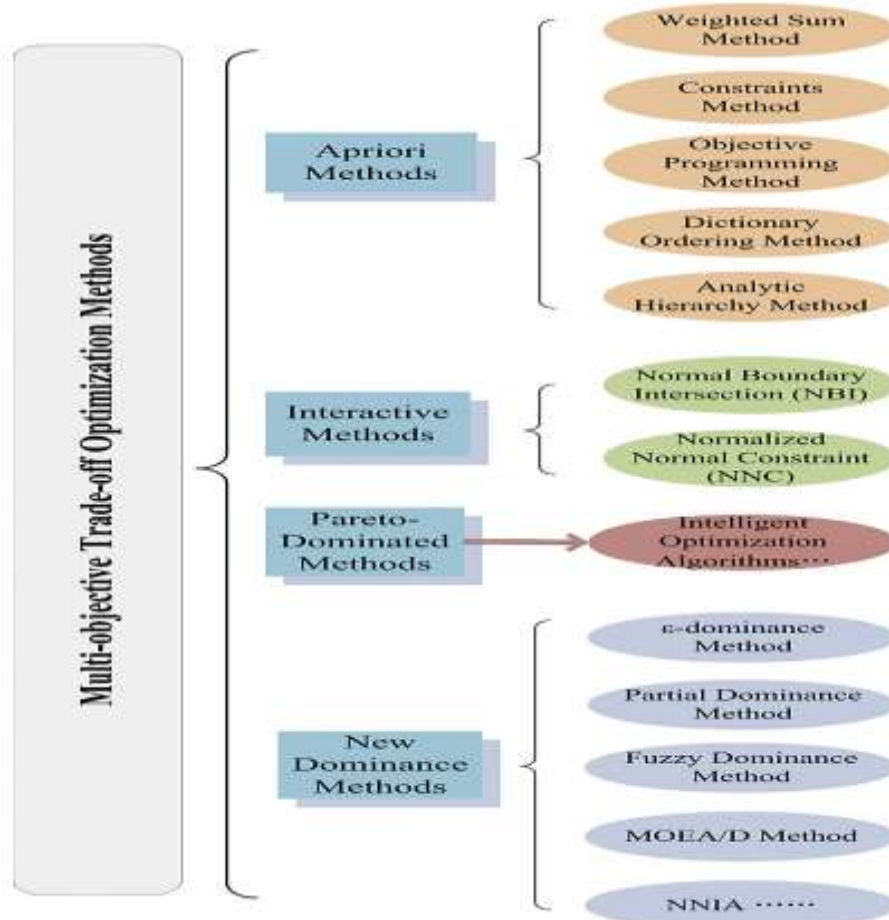


Fig. 5. Classification of multi-objective trade-off optimization methods

3. Interactive methods

In order to actively exploit the decision makers' knowledge and experiences, interactive methods were developed, such as the interactive weighted Tchebycheff method [74], the Light Beam Search [75] and the NIMBUS method [76, 77]. The interactive methods incorporate preferences of decision makers for each objective during the optimization process. As usual, interactive methods implement an achievement scalarization function (ASF) to generate Pareto optimal alternatives. There are two widely-applied interactive methods for dealing with multi-objective decision problems: Normal Boundary Intersection (NBI) and Normalized Normal Constraint (NNC) methods. They are relatively new scalarization methods compared with the WSM, which reformulate the multi-objective optimization problem into a parametric single objective optimization problem. In 1998, Das and Dennis [78] proposed the NBI method, which tackled the multi-objective problems from a geometrically intuitive viewpoint. The method first builds the convex hull of individual minima (CHIM) and then constructs (quasi-)normal lines to the plane. The rationale lies in that the intersection between the (quasi-)normal from any point on the CHIM, and the boundary of the feasible objective space closest to the origin is expected to be the Pareto optimal. The NBI method is able to form a near-uniform spread of the Pareto-optimal frontier, making the NBI a more attractive approach to the Weighted Sum Method in solving non-convex, high-dimensional multi-objective problems. Ganesan et al. [79] used the NBI interactive method to compromise the multiple optimized objectives in the synthesis gas production process of combined carbon dioxide reforming and partial-oxidation of methane technologies. In conjunction with the NBI method, the GSA and the PSO algorithms were adopted to realize the process optimization of objectives of methane conversion, carbon monoxide selectivity and the hydrogen to carbon monoxide ratio. The optimization results of these two algorithms were compared using the Euclidean distance metric.

The PSO algorithm outperformed the GSA method in terms of uniformity of the Pareto front and computational efficiency. In 2003, Messac et al. [80] proposed the NNC method, which was similar as the NBI method but combined with features of the ε -constraint method. The ε -constraint method minimizes the most important objective functions, while the other objectives are added as inequality constraints with the form $E \leq E$. Based on this idea, in the NNC method, a plane called the utopia hyperplane is constructed through all normalized individual minima, and equally distributed points in this hyperplane are determined by consistently varying the weights. Then $m-1$ hyperplanes are constructed for other objective functions. These hyperplanes are chosen perpendicular to each of the $m-1$ utopia plane vectors, which join the individual minimum corresponding to the selected objective functions. Furthermore, there is an Enhanced Normalized Normal Constraint method (ENNC) proposed in Ref. [81]. Logist et al. [82] incorporated the NBI and NNC methods in a deterministic multiple shooting optimal control to mitigate the drawbacks of the Weighted Sum Method. The combined interactive method was able to obtain an equal distribution along the non-convex Pareto front. It can deal with equality/inequality constraints and boundary value problems, with tight tolerances for global and local optimality. The resulting optimization method is successfully used in the design of a chemical reactor and the control of a bioreactor. The integration of optimization techniques with these interactive methods were efficiently used to tackle non-convex optimal control problems [83] and applied to different engineering fields [145].

However, NBI and (E)NNC may overlook the extreme parts of the Pareto set. Therefore, Vallerio et al. [84] introduced an Interactive Geometric Extension (IGE) technique to extend the Pareto set for NBI and (E)NNC methods based on geometric considerations. Then the extended NBI or (E)NNC methods were applied successfully to three scalar multi-objective problems and the multi-objective optimal control of a tubular and a fed-batch reactor. The results demonstrated the low computational burden and applicability to higher than three dimensional problems of the proposed methods. Moreover, Vallerio et al. [85] presented an interactive framework based on NBI and ENNC to realize the nonlinear dynamic multi-objective optimization. By the active use of Pareto Browser Graphical User Interface (GUI), decision makers expressed their preferences via the browsing of scalarization parameters such as weights. The parameters were adapted interactively. Finally, the introduced interactive framework for multi-objective dynamic optimization was successfully tested for a three and five-objective fed-batch reactor case study with uncertain feed temperature and heat transfer parameters. Most interactive methods for multi-objective optimization problems may impair at least one objective function to get a solution. Hence, Miettinen et al. [86] proposed a NAUTILUS method based on the assumptions that past experiences affected decision makers' hopes and decision makers did not react symmetrically to losses and gains. The ability of NAUTILUS to obtain a non-anchored Pareto optimal solution made it an ideal tool to find an initial solution for any other interactive schemes. In order to deal with the objectives impairment problem, Bortz et al. [87] integrated an algorithm based on a state-of-the-art steady-state flow sheet simulator for designing a distillation process for the separation of an azeotropic mixture. Firstly, a minimal Pareto set with predefined accuracy was calculated by the sandwich approximation method, which can handle non-convexities. Then the decision makers navigated interactively on the Pareto set and explored different optimal solutions by the CHEMASIM tool.

III. CONCLUSION

In this paper, the description of multi-objective optimization problems and solutions definition is given in summary. Due to the complexity, orthodox and mostly nonlinearity of multi-objective optimization problem, intelligent optimization algorithms like evolution based and swarm based algorithms were proposed and has been improved continuously for solving the problem with good performance. We also give a brief introduction of some most often used intelligent optimization algorithms about their development processes and (dis)advantages. Some existing test problems consisted of mathematical functions are also demonstrated and the relative performance indexes for verifying the effectiveness of multi-objective optimization methods are summarized. In order to get a final optimal solution for specific multi-objective problems, trade-off optimization methods including a prior methods, interactive methods, Pareto-based methods and new dominance methods were proposed and improved.

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